"Either-Or" Two-Slit Interference: Stable Coherent Propagation of Individual Photons Through Separate Slits

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ABSTRACT In quantum theory, nothing that is observable, be it physical, chemical, or biological, is separable from the observer. Furthermore, ". . . all possible knowledge concerning that object is given by its wave function" (Wigner, E. 1967. Symmetries and Reflections. Indiana University Press, Bloomington, IN), which can only describe probabilities of future events. In physical systems, quantum mechanical probabilistic events that are microscopic must, in turn, account for macroscopic events that are associated with a greater degree of certainty. In biological systems, probabilistic statistical mechanical events, such as secretion of microscopic synaptic vesicles, must account for macroscopic postsynaptic potentials; probabilistic single-channel events sum to produce a macroscopic ionic current across a cell membrane; and bleaching of rhodopsin molecules (responsible for quantal potential "bumps") produces a photoreceptor generator potential. Among physical systems, a paradigmatic example of how quantum theory applies to the observation of events concerns the interactions of particles (e.g., photons, electrons) with the two-slit apparatus to generate an interference pattern from a single common light source. For two-slit systems that use two independent laser sources with brief ($<$1 ms) intervals of mutual coherence (Paul, H. 1986. *Rev. Modern Phys.* 58:209–231), each photon has been considered to arise from both beams and has a probability amplitude to pass through each of the two slits. Here, a single laser source two-slit interference system was constructed so that each photon has a probability amplitude to pass through only one or the other, but not *both* slits. Furthermore, all photons passing through one slit could be distinguished from all photons passing through the other slit before their passage. This "either-or" system produced a stable interference pattern indistinguishable from the interference produced when both slits were accessible to each photon. Because this system excludes the interaction of one photon with both slits, phase correlation of photon movements derives from the "entanglement" of all photon wave functions due to their dependence on a common laser source. Because a laser source (as well as Young's original point source) will have stable time-averaged spatial coherence even at low intensities, the "either-or" two-slit interference can result from distinct individual photons passing one at a time through one or the other slit–rather than wave-like behavior of individual photons. In this manner, single, successive photons passing through separate slits will assemble over time in phase-correlated wave distributions that converge in regions of low and high probability.

INTRODUCTION

Quantum theory has been considered as a theory of knowledge that concerns all of science, extending from physics to chemistry, biology, and perceptual psychology. Statistical mechanical phenomena in biophysical systems for which quantum theory has particular relevance include probabilistic secretion of quanta (Bennett et al., 1997) at synaptic junctions, single photon responses of rods (Baylor et al., 1979) and *Limulus* photoreceptors (Fuortes and Yeandle, 1964), and single-channel events in membrane "noise" (De Felice, 1981). A fundamental tenet of quantum mechanical theory holds that no observable object, be it physical or biological, is entirely separable from the observing apparatus or the observer. Thus, the observed object, the observing apparatus, and the observer are all parts of a system whose interactions must be taken into account in any description of that object. A second tenet, as formulated by Wigner (1967), states that "Given any object, all possible knowledge concerning that object can be given as its wave function . . . the wave function permits one to foretell with what probabilities the object will make one or another impression on us if we let it interact with us either directly, or indirectly." With such wave functions, according to Bohm (1989), " . . . we can treat all systems, however complex, and get quantitatively correct results in a tremendous number of applications. . . ." A third tenet of quantum theory describes an unavoidable uncertainty that is intrinsic to the object-observer system. The observation process itself alters what is observed and limits the completeness of the knowledge derived.

The paradigmatic example of how quantum theory applies to the observation of events has been provided by the observed interactions of particles (e.g., photons or electrons) with the two-slit apparatus. Propagation of photons through a two-slit system, originally described by Young in 1801 (cited in Bohr, 1949), can be described by a wave function whose square is proportional to the probability of observing a photon at the two-slit target screen. This probability function includes 1) phase factors to describe the wave-like oscillations of the photon probability distributions; and 2) interference terms to describe the likelihood of

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photon occurrence at points within the two-slit pattern for photons that move through both slits.

A critical requirement for two-slit interference is the phase correlation for photon movements from one slit with photon movements from the other slit. As stated by Bohm (1989), "As long as definite phase relations between the wave function for slit A and the wave function for slit B exist, the electron (or photon) is capable of demonstrating the effects of interference and acting as if it passed wave-like through both slits simultaneously." Furthermore, ". . . interference terms . . . would not be present if the experiment involved a probability distribution of classical particles, coming either through slit A or slit B." Emphasizing the latter point, Feynman et al. (1965) claim "It is not true that the electrons (photons) go either through hole 1 or hole 2."

The two-slit interference pattern for photons or subatomic particles such as electrons (cf. Tonomura et al., 1989) is the same when generated by an intense source or by a dim source that transmits one particle at a time (Taylor, 1909; Feynman et al., 1965). Thus, the particles need not interact with other particles, and it has been inferred must, therefore, interact individually with a two-slit system to produce two-slit interference. Each individual particle must interact with *both* slits or, as has been described (Dirac, 1947), "Each photon interferes only with itself. Interference between two different photons never occurs." Unlike a photon's localized interaction with a silver grain on a photographic plate or a rhodopsin molecule during biological transduction, a photon has been thought to interact nonlocally or in a wave-like manner with the two slits. The nature of the measuring apparatus, silver grains versus a two-slit system, determines the nature of detection: particle versus wave (Bohr, 1935). Thus, the required phase correlation for two-slit interference (see above) has been considered to arise from each individual photon's interaction with both slits.

These inferences about two-slit interference were first applied to thermal light sources that required a beam-splitter (e.g., a half-silvered mirror or a screen hole) to distribute a common primary beam across the two slits. Subsequently, Magyar and Mandel (1963) showed that two independent laser beams produced transient ($<$1 ms) two-slit interference when intervals of coherence were detected (see also Mandel, 1964; Basano and Ottonello, 2000). This detection, however, also required the use of beam-splitters and, in Mandel's words, "the positions of the fringe maxima and minima are unpredictable by definition of incoherent beams." Still later, Radloff (1971) used a feedback-shutter control system to reinforce a reference two-slit interference pattern with two low-intensity laser beams over a 30-min interval. In these and other systems it was never possible to determine, however, from which independent laser beam a given photon arose. In these previous experiments with two independent lasers, therefore, the transience of the correlation and the use of beam-splitters precluded questions about "self-interference." According to Bohr (1949), "If a semi-reflecting mirror (or another beam splitter) is placed in the way of a photon . . . (we are) obliged to say, on the one hand, that the photon always chooses *one* of the two ways (directions of propagation), and, on the other hand, that it behaves as if it had passed *both* ways." In Paul's description (1986), "What actually happens in that detection process is that an energy packet $h\nu$ is taken from the superposition field to which both lasers contribute equally." In an alternative description of all of these two-slit systems (Feynman, 1985), each photon would have a probability amplitude to arise from *either* independent laser beam before moving through one of the two slits.

With an experimental design modeled after Taylor's original 1909 experiment, each photon, moving one at a time from a coherent source, has a probability amplitude to move through one or the other of the two slits in a two-slit interference apparatus. Such a one-photon-at-a-time experiment would not test the possibility, however, that the same two-slit interference would result if each of the two slits could only open separately (i.e., without the other slit open), randomly, and for equal average durations over the entire observation interval. For such conditions in a one-photon-at-a-time two-slit experiment, each photon would have a probability amplitude to move through one open slit, but not the other closed slit.

Here, we will construct a two-slit system in which each and every individual photon can only pass through one or the other, but not both slits. Thus, for each photon, only one slit will be open and one slit will be closed. Photons, then, with a probability amplitude to move through one slit will have no probability amplitude to move through the other slit, and vice-versa. In past approximations of such an "either-or" two-slit system, division of a coherent light source by a beam-splitter provides two "independent" coherent sources, but each source is composed of photons that have probability amplitudes to move through either slit.

In the complete system constructed here (Fig. 1), no photons have access to both slits. Therefore, a particular photon that is accessible to one slit will not be accessible to the other slit, and vice-versa. Thus, the interference terms for the resulting "either-or" two-slit pattern (see Results) cannot be ascribed to photons passing through both slits, but must be explained by stably coherent movements of different photons through separate slits. Because the laser source and Young's point source will remain spatially coherent at low intensities, and the coherent distributions of these photons remain stable over time, the "either-or" nature of this two-slit system can apply to passage of photons one at a time over prolonged observation intervals. Thus, as Mandel (1964) suggested for transiently coherent "independent" sources, "In principle at least, the result of the experiment should be unchanged if on the average only one photon at a time were to traverse the interferometer."

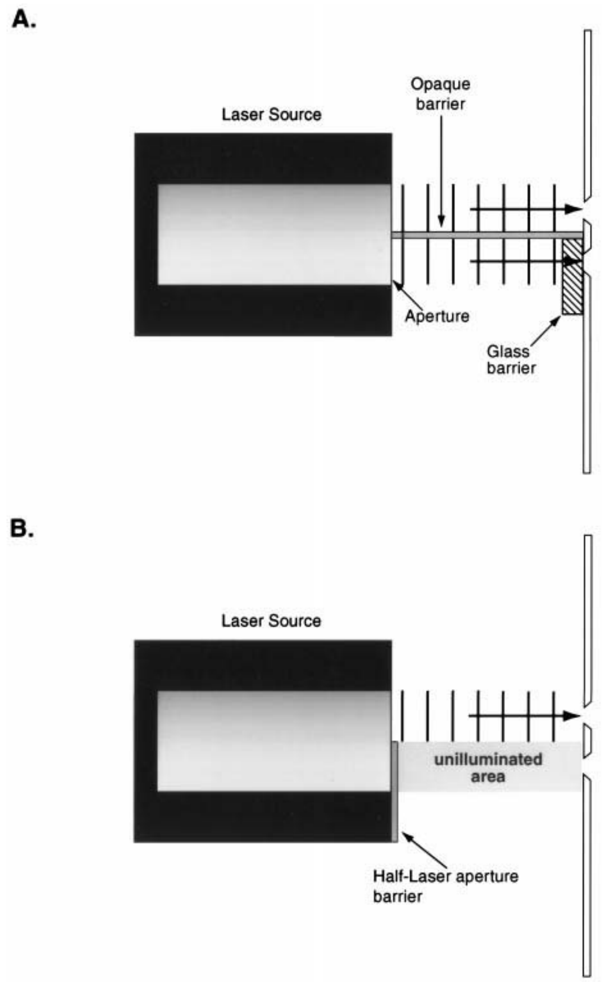

FIGURE 1 Relation of planar waves to the laser source with an 0.25-mm opaque barrier bisecting the laser beam, i.e., in an "either-or" configuration (*A*) and with a film barrier over one-half of the laser aperture (*B*). The dashed rectangle indicates a glass barrier placed in immediate apposition to one slit.

METHODS

To construct such an "either-or" two-slit system, a completely opaque barrier, consisting of black film, was positioned throughout the distance between a laser source (670 nm diode laser, 3.0 mW, 2 mm diam) and the wall separating the two-slit apertures (Fig. 1). The target screen was 0.5 m from a condensing lens (4×, Nikon objective). This completely opaque film was ~0.25 mm thick, while the distance between the two slit apertures, as well as the width of the apertures themselves, was ~0.5 mm. One end of the 2-cm-long film barrier was in close apposition to the wall separating the two slits, while the other end was in close apposition to the laser source aperture. Other configurations were also tested. In one, the two-slit screen itself was placed in direct apposition to the laser aperture. In another configuration, the two-slit screen (without the barrier) was placed 1 mm away from the laser aperture. In still another configuration, the film barrier (leading to the two slits, as in Fig. 1) was placed 1 mm away from the laser aperture.

Finally, an important additional element (see Unilateral Glass Barrier below) of the system was then added to ensure (see below) the "either-or" nature of this two-slit system. With the opaque barrier dividing the laser beam, a glass barrier (0.2–1.0 cm) was placed in immediate apposition to one of the two slits (Fig. 1).

When present, the film barrier divided the laser beam exactly into two equal right and left sections. The laser amplification process itself will now be discussed to clarify the movements of photons within and from the laser cavity.

Laser source

In contrast to photons from an isotropic source, photons emitted from a laser source move with almost identical wave vectors along paths that are parallel to the long axis of the laser resonating cavity. Within the laser cavity, only photons moving along the cavity axis will stimulate successive atoms to emit photons that also move along this long axis. In this way, the laser amplification process selects (or "chooses") the preferred direction of photon propagation (Townes, 1960). In the diode laser source used here, many successive photons with the same vectors require many successive reflections from the reflecting surfaces at each end of the laser. Within the laser cavity resonator, stimulated emission effectively minimizes the probability amplitudes for photon movements that are not parallel to the long axis of the laser cavity.

Photon movements immediately before exiting the laser aperture, therefore, are unidirectional with negligibly few exceptions, and are completely coherent, both with each other and with movements of photons immediately after emission from the laser aperture. Thus, there should also be negligibly few deviations in the directions of photon movements immediately beyond the laser aperture.

Divergence of laser light

For a unidirectional, spatially coherent laser beam, divergence in the near field results only from diffraction effects at the laser aperture (Milonni and Eberly, 1988). Thus, divergence of a spatially coherent laser beam in the near field can be termed "diffraction-limited." Considering only these diffraction effects, the movement of a particular photon emerging from the laser aperture will diverge from the laser's long axis vector by an angle that is approximately equal to the photon's wavelength divided by the aperture diameter (McGervey, 1995). For a 600-nm wavelength and 1-mm aperture, the divergence is ~0.6 milliradians. At a distance of 1 cm or less, therefore, the beam size has increased by <1%. Divergence of the diode laser beam used here was measured for the near field (1–2 cm) from the lateral projection of the laser beam on the black film barrier. With the barrier exactly bisecting the laser beam (Fig. 1 *A*), the illuminated beam projection on the barrier could be visualized and measured for the full length of the barrier. The height (which is equal to the 2-mm diameter of the beam at its origin) of this projected image remained a constant 2 mm along the entire length of the barrier. Thus, there was no measurable change in the beam diameter and, therefore, no measurable divergence of the beam for 1- 2-cm distances from the aperture. Negligible divergence was further confirmed for the laser source used here by placing a black film barrier across one-half of the laser source at its emitting aperture (Fig. 1 *B*). The edge of this barrier projected on a planar target screen (2 cm or less from the laser aperture) with a sharply defined border between the illuminated and darkened halves of the laser beam. With the half-aperture barrier in place only one slit was illuminated, while the other slit was in complete darkness. Thus, no detectable light diverged from the non-obstructed side across the distance from the laser aperture to the target screen. In a final test of divergence, the barrier was placed in the center of the beam path but not in direct apposition to the slits. Under these conditions, a distinct darkened shadow appeared between the two slits, each of which was still illuminated by one-half of the laser beam. Thus, the divided halves of the laser beam do not diverge to the opposite side, i.e., across the 0.25-mm darkened band between them.

Divergence of single photons at the barrier edges

To divide wave-like at the edges of the barrier that divides the laser beam, a single photon would have to "see" each edge of the barrier. Stated in another way, a given photon would have to move along possible paths past each edge of the barrier. Wave-like division of a single photon at the barrier edges, therefore, would require significant probability amplitudes for a single photon's movements that span the thickness of the barrier, i.e., 0.25 mm. Paths of a single photon that span this distance are not consistent, however, with the measured lack of divergence (see above) in the near field for the spatially coherent laser beam used here. Nor are such paths that span the 0.25-mm-thick barrier consistent with the predicted lack of divergence (i.e., much less than an 0.01 mm increase of beam diameter) at 1 mm from the laser aperture.

Based on laser divergence considerations discussed above, the collimated, coherent laser light beam that emerged from the laser housing traveled separately either on the right or left side of the barrier in the either-or two-slit configuration used here. Individual photons issuing from the right side of the laser aperture, therefore, would only enter the right-sided slit, but not the left slit. Similarly, individual photons issuing from the left side of the laser aperture could only enter the left-sided slit, but not the right slit.

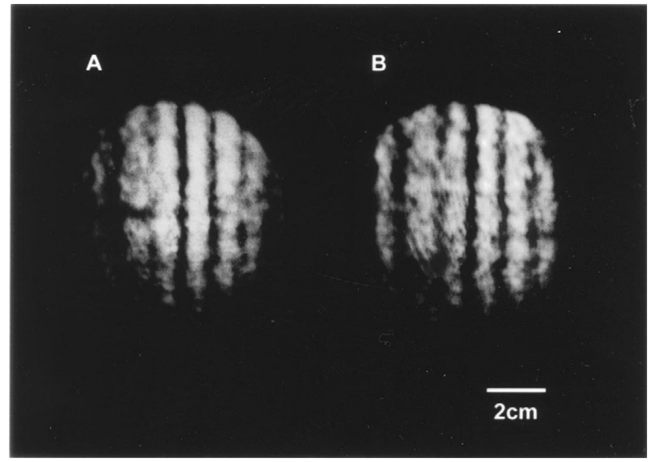

FIGURE 2 Interference produced by the two-slit system when (*A*) no barrier was present, and (*B*) barrier in place. Note the close similarity of the two-slit dark and light bands for both conditions.

A unilateral glass barrier

Notwithstanding the considerations of divergence just discussed, it still might be argued that the nonlocalized nature of photons, i.e., the transverse extension described by the wave function, could still allow each photon to move along both sides of the opaque barrier toward each of the two slits. To rule out such a possibility, a glass barrier (barriers of thickness varying between 0.2 and 1.0 cm thickness were tested) was placed immediately adjacent to only one of the two slits, i.e., on one side of the opaque barrier that divides the laser beam (cf. Fig. 1). Based on considerations of light scattering (cf. Feynman, 1985), the laser photons entering the glass barrier are absorbed by electrons in the glass, and different photons emitted by electrons in the glass exit the glass to pass through the abutting slit. This configuration ensures, therefore, that the photons exiting the glass and passing through the abutting slit have no possibility of gaining access to the other slit on the other side of the opaque barrier. Similarly, laser photons moving through air have no access to the slit behind the glass barrier. Thus, photons can only pass through one or the other, but not both, slits. This forced "either-or" two-slit configuration allows certain photons from the glass to have a probability amplitude to move through one slit (i.e., behind the glass), and allows *different* photons from the laser to have a probability amplitude to move through air, and then the other slit.

RESULTS

With no barrier in place, the two-slit system produced an interference pattern (Fig. 2 *A*) that was virtually the same classical pattern, with alternating light and dark bands, as has been observed in countless previous studies. With the barrier in place (as in Fig. 1 *A*), the two-slit system produced an essentially identical interference pattern (Fig. 2 *B*) as that produced without the barrier. The presence of the completely opaque barrier, therefore, did not alter the two-slit interference pattern. This same two-slit pattern was also obtained when the laser aperture itself was in direct apposition to the two slits. In this limiting case, in which divergence was negligible, the thickness of the barrier between the two slits was effectively reduced to the thickness of the two-slit screen itself. A third case was tested in which one end of the barrier was positioned 1 mm away from the laser aperture while the barrier's other end was closely approximated to the screen portion between the two slits. In this third case, therefore, the laser beam had already exited the laser aperture, showed negligible divergence, and consisted almost entirely of photons with identical wave vectors distributed over either the right or left side of the laser beam. In all of these cases, essentially the same two-slit interference pattern was obtained. In the final, most complete configuration both an opaque barrier bisecting the laser beam and glass barrier in front of only one slit were present. The characteristic two-slit interference pattern (cf. Fig. 2) was entirely preserved for a glass barrier whose thickness ranged from 0.2 to 1.0 cm—notwithstanding the loss of 4% light reflected by the glass barrier.

DISCUSSION

As photons leave the laser aperture, negligible divergence occurs in the near field, i.e., immediately adjacent to the aperture. Furthermore, a lack of significant divergence for small distances (e.g., 1–20 mm) from the laser aperture was demonstrated (see Methods) when a film barrier over one-half of the laser aperture caused the absence of light between this half-aperture barrier and the slit on the same side (cf. Fig. 1 *B*). Thus, the probability amplitudes for photons on one side of the laser beam to move to the other side (immediately *after* the aperture) should not be significant, nor are these probability amplitudes consistent with photon movements that span the 0.25 mm thickness of the barrier.

It may still be argued, however, that the transverse extension of a photon's wave function allows it to move down both sides of the opaque barrier toward each of the two slits. In such a case, the conditions of a Taylor-type experiment whereby each photon extends across both slits would still obtain. To unequivocally rule out this latter possibility, a glass barrier was placed adjacent to only one slit with the opaque barrier bisecting the laser beam still in place. This configuration, which still produced characteristic two-slit interference, ensured that no photons would have a "choice" of (or probability amplitudes for) both slits (see Methods—Unilateral Glass Barrier).

The probability of arrival for a particle at the target screen can be considered (cf. Bohm, 1989; Feynman et al., 1965; Hecht, 1998) as the square of a complex number, Φ, called the probability amplitude. The phase of the probability amplitude is a function of space and time. For each particle, Φ is the sum of the Φ values for all possible paths from a source to the target screen. Thus, for the two-slit system's probability function, P,

$$P = |\Phi_1 + \Phi_2|^2$$

where Φ_1 is the probability amplitude for slit 1 and Φ_2 is the probability amplitude for slit 2. For particles arriving at the target screen *in phase*,

$$P = |\Phi_1 + \Phi_2|^2 = 4|\Phi_1|^2.$$

For particles arriving at the screen completely *out of phase*,

$$P = |\Phi_1 - \Phi_2|^2 = 0.$$

The phase correlation between photon movements emanating from one slit and photon movements from the second slit critically determines the interference pattern generated by a two-slit system. Because the same two-slit interference results when photons pass through the system one at a time, it has been inferred that this critical phase correlation arises from the quantum mechanical interaction of individual photons with *both* slits. According to this inference, the wave nature of each individual photon allows it to move along many paths "simultaneously," thereby requiring a probability amplitude that takes into account all possible paths and is, therefore, the sum of the probability amplitudes for all possible paths. This same wave nature of individual photons has been invoked to explain why the two-slit probability function (or the intensity distribution) at the target screen is not equal to the sum of the probability functions for each of the two slits.

In the "either-or" two-slit system (Fig. 1), photons do not have a choice of two slits, but there is, nevertheless, phase correlation (i.e., spatial coherence) of photon movements from the left side of the laser with photon movements from the right side of the laser—even for the different photons exiting the glass barrier. In the "either-or" system, therefore, there appear to be at least three requirements to produce two-slit interference: 1) the structural configuration of a two-slit system; 2) simple addition of photons passing through each slit separately; and 3) the photon movements through the separate slits must be phase-correlated—i.e., spatially coherent. This spatial coherence must apply even to the wave distributions of successive photons that move one at a time through the two-slit system.

The "either-or" two-slit pattern, therefore, does not include probability amplitudes for paths through *both* slits, yet the product interference term within the two-slit interference probability function still accurately describes the vector cancellation of out-of-phase photons and addition of in-phase photons. The implication follows, then, that for "either-or" two-slit interference the interference term need not arise from phase correlation of the *same* individual photon's passage along multiple paths (i.e., through both slits). *Different* photons, instead, pass with phase-correlated movements through separate slits to produce interference. Each of these photons has probability amplitudes to move through one or the other, but not both, of the two slits. Furthermore, we know the source of all photons for one slit (the laser), and that this source is separate from the source of virtually all photons from the other slit (the glass barrier itself).

Time-averaged phase correlation of *different* photons, passing without a "choice," through separate slits of the "either-or" system generate detectable regions of low probability and high probability at the target screen. It is this phase correlation that changes the simple addition of each slit's intensity distribution (i.e., probability function) into the alternating maxima and minima of the "either-or" two-slit interference pattern.

Furthermore, the implication of "either-or" interference for different photons' correlated movements through each slit may be extended to a Taylor-type experiment in which two-slit interference is produced by passage of one photon at a time through a two-slit system. Since incoherent atomic radiators that are not emitting photons simultaneously still produce spatially coherent light through a point source, we can infer that time-averaged spatial coherence will be maintained for photons passing one at a time through the two-slit system. Because the coherent, but separate, photon ensembles move through separate slits in distributions that are stable over time, we might predict that the characteristic two-slit pattern would result from successive photons passing one at a time through the "either-or" system. Provided that time-averaged rate of passage of phase-correlated photons one at a time is equal for each slit, the accumulated interference pattern should result in the "either-or" two-slit system.

Although the Taylor-type experiment (1909) with passage of one photon at a time demonstrated that interaction of photons is not required for two-slit interference, it did not rule out that each photon might "see" only one slit at a time and still produce the two-slit pattern. Such a possibility, as now indicated by the "either-or" interference above, can also be tested with an oscillating shutter. An oscillating shutter would alternately open one or the other slit during the passage of photons through the two-slit system. Although low-frequency oscillation smeared the pattern, the latter being previously described (cf. Feynman et al., 1965), high-frequency shutter oscillation might reveal a classic two-slit pattern.

As emphasized in the above discussion, photons from the glass barrier cannot interact with photons or the slit on the other side of the opaque barrier. Similarly, laser photons entering one slit cannot interact with the other slit behind the glass barrier. Thus, the "either-or" system prevents "action-at-a-distance," whereby each photon interacts with both slits. Instead, the wave functions of all photons are "entangled" due to their origin from a common laser source.

Detection of "either-or" two-slit interference, therefore, implies that wave distributions of particles (cf. Born, 1989; Schrödinger, 1928) passing through one slit separately can add to phase-correlated wave distributions of different particles passing through the second slit separately to cause "destructive" or "constructive" interference. Because of their phase-correlated passage through separate slits, individual photons are not interacting, but instead are *assembling* in regions of low probability (destructive interference) and in regions of high probability (constructive interference). The wave-like division of individual particles (or "self-interference") cannot occur immediately before or at the slits in the "either-or" two-slit system used to produce interference. Thus, considering the "either-or" system results above, two-slit interference need not necessarily imply the wave-like behavior of individual particles, but does require the phase-correlated wave distributions of many particles (Born, 1989; Schrödinger, 1928). The demonstration here of two-slit interference with the "either-or" system raises the possibility for the first time, therefore, that all previous two-slit interference patterns can be explained without invoking "self-interference" or wave-like extension of individual photons across both slits.

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